

Program Title SEQUENCESContributor's Name Mondecai Schwartz, M.D.Address 119 Hewlett Neck RoadCity Woodmere, L.I.State N.Y.Zip Code 11598

Program Description, Equations, Variables *The program generates a diverse and interesting mix of relatively obvious, moderately difficult, and exceptionally challenging integer sequences. Many thousands of sequences in hundreds of categories are randomly selected and presented.*

*The user is challenged to determine a generating pattern for each sequence and to predict further terms. Many sequences may have several seemingly different yet operationally equivalent modes of evolution.*

*Each integer sequence is uniquely determined by its category constants, generating constants, and placeholder (for 1<sup>st</sup> two terms) constants. The sequences themselves are randomly presented, so that you never know what may be coming next.*

*If you are unable to determine a generating pattern for a particular sequence, its <sup>category</sup> generating constants may be recalled for a clue. These integer constants - a, b, c - refer to components in the corresponding storage registers. The sequence category is  $(Ra \cdot Rb + Rc)$ .*

## Operating Limits and Warnings

*The user may at any time repeat his last sequence or go on to the next. Sequences of particular interest may be recorded and coded by their generating seeds for later re-generation. On the HP-97, successive sequences may be recorded and formatted along with their generating seeds and category constants.*

This program has been verified only with respect to the numerical example given in Program Description II. User accepts and uses this program material AT HIS OWN RISK, in reliance solely upon his own inspection of the program material and without reliance upon any representation or description concerning the program material.

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Sketch(es)

The integer sequences below represent the first 10 sequences generated from initial seed 0.123456789. For each sequence, the generating seed and the category constants  $a, b, c$  are shown.

Should you be unable to solve any particular integer sequence, you may use its category constants to obtain the generating formula  $Ra \cdot Rb + Rc$ , where, e.g. if  $a=7$ ,  $Ra$  refers to the sequence component in  $R7$ . See p. 4 for register assignments.

Sample Problem(s)	Sequence	Seed	<u>a</u>	<u>b</u>	<u>c</u>
①	3 8 14 16 <sup>18</sup> 19 19 21 26 32 <sup>36</sup> 37 37...	0.123456789	9	7	18
②	-1 1 5 11 19 29 41 55 71 89 109...	0.704423990	11	8	1
③	-3 3 -7 7 -11 11 -15 15 -19 19...	0.347521770	2	16	1
④	-4 -3 -2 -1 0 1 2 3 4 5 6...	0.039254880	1	10	3
⑤	0 5 8 17 24 37 48 65 80 101...	0.075044970	4	2	19
⑥	-5 -7 -9 -11 -13 -15 -17 -19...	0.682984950	8	3	4
⑦	-2 2 3 -1 -2 2 3 -1 -2 2...	0.984957340	0	0	15
⑧	-1 1 -1 1 -1 1 -1 1 -1 1...	0.915131950	6	16	14
⑨	-4 5 -6 3 -8 1 -10 -1 -12...	0.054253710	2	4	10
⑩	-2 -1 1 0 -1 1 -2 -1 1 0 -1...	0.012933370	13	12	2

Solution(s)

For each sequence of integers try to discern the generating formula or pattern, and predict further members of the sequence. Some sequences are elementary and obvious and others quite challenging and deep. In many instances, even though a particular formula generates the sequence, you may perceive a seemingly different yet operationally equivalent pattern for evolving further terms.

Reference(s)

Should a particular sequence remain intractable even after utilizing the category constants, you may as a last resort obtain the specific generating constants  $k_1, k_2, k_3$  (see register assignments).

SEQUENCES			
1	(RND $\Phi$ )	2	
NEW S	LAST S	GO	SEE n
AUTO ? SEE a, b, c			

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load both sides of program card			
2	Select +/-a display initial seed If no new seed is selected, current seed is displayed. Initial default k=0	$0 \leq k \leq 1$	f a	k
3	Prepare for a new sequence		A	} pause/print generating seed
3'	Prepare for repetition of last sequence		B	
	Choose either 3 or 3'. The generating seed in the display/printed identifies the particular sequence, and at any time may be directly loaded (via step 2) to reproduce that sequence.			
4	Choose type of output desired		f d	} 1= AUTO 0= single
5	Generate the sequence terms		D	
	Press D repeatedly in single-term mode. Press once only in auto mode. After 5 or more terms are pause/printed, try to determine the law of formation or generating pattern. Predict and confirm further terms.		(D)	$a_1$
			(D)	$a_2$
			(D)	$a_3$
			(D)	$a_4$
			:	:
6	At any time (single-term mode), see no. of terms generated to date		E	n
7	Obtain sequence-category clues, if needed		f e	a, b, c
	Constants a, b, c (integers) refer to components in the corresponding R registers. Sequence formula = $R_a \cdot R_b + R_c$			(pause/print)
8	For a new sequence go to step 3 To repeat last sequence go to step 3'			

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
001	g LBL 0	22 25 11	Change %a display current RAND. NO. generator seed for next sequence.		STO 8	23 08	Start initialization  1 → R <sub>0</sub> -1 → R <sub>1</sub> n = 0 → R <sub>2</sub> (-1) <sup>n</sup> = 1 → R <sub>2</sub>
	h F? 3	35 71 03			1	01	
	STO E	33 15			STO 0	33 00	
	RCL E	34 15		060	STO 2	33 02	
	DSP 9	23 09			CHS	42	
	h RTN	35 22	Option for con- tinuous generation + display/printout of sequence ele- ments.  F <sub>0</sub> ON → 1 = AUTO F <sub>0</sub> OFF → 0 = single A		STO 1	33 01	Obtain fixed + diphasic constants  k <sub>1</sub> → R <sub>3</sub> k <sub>2</sub> → R <sub>4</sub> -k <sub>2</sub> → R <sub>5</sub>  (-1) <sup>n</sup> k <sub>2</sub> = k <sub>2</sub> → R <sub>6</sub> } n=0 (-1) <sup>n</sup> k <sub>1</sub> = k <sub>1</sub> → R <sub>7</sub> }
	g LBL 1	32 25 14			f GSB 1	31 22 01	
	DSP 0	23 00			STO 3	33 03	
	h F? 0	35 71 00			STO 7	33 07	
010	GTO 4	22 04			f GSB 1	31 22 01	
	h SFO	35 51 00	Display/printout of sequence cate- gory constants a, b, c of the sequence (a) · (b) + (c)  (a) = component in R <sub>a</sub> etc.		STO 4	33 04	Complete the initialization  k <sub>3</sub> - n = k <sub>3</sub> → R <sub>50</sub> n - k <sub>2</sub> = -k <sub>2</sub> → R <sub>51</sub>  a <sub>n-1</sub> → R <sub>52</sub> } place- a <sub>n-2</sub> → R <sub>52</sub> } holds
	1	01			STO 6	33 06	
	h RTN	35 22			f GSB 1	31 22 01	
	f LBL 4	31 25 04		070	CHS	42	
	h CFO	35 61 00			STO 5	33 05	
	0	00	Display ordinal no. n of current se- quence element a <sub>n</sub>  B repeats the last sequence by placing its generating seed into R <sub>5</sub> . A uses (and stores in R <sub>4</sub> ) R <sub>5</sub> seed.		f PLS	31 42	if $\begin{cases} a \leq 7 \\ b \leq 7 \\ c \leq 7 \end{cases}$  are all true, adjust to avoid a degenerate sequence.  c' = c + 12  Also, if: a ≤ 7 set F <sub>1</sub> b ≤ 7 set F <sub>2</sub> for later a', b' adjustment.
	h RTN	35 22			STO 1	33 01	
	g LBL E	32 25 15			CHS	42	
	DSP 2	23 02			STO 0	33 00	
020	h space	35 84			f GSB 2	31 22 02	
	RCL A	34 11	Generate the sequence category constants a, b, c (integers), and store in R <sub>A</sub> , R <sub>B</sub> , R <sub>C</sub> .  0 ≤ a ≤ 13 0 ≤ b ≤ 16 0 ≤ c ≤ 19  0-19 correspond to R <sub>0</sub> -R <sub>19</sub> via indirect addressing.		STO 2	33 02	if a %a b cor- respond to a sequence component which is a constant or bi- phasic constant = 0, then adjust a %a b to alter the sequence category.  a' = a + 3 b' = b + 3
	f -X-	31 84			f GSB 2	31 22 02	
	RCL B	34 12			STO 3	33 03	
	f -X-	31 84		080	f PLS	31 42	
	RCL C	34 13			7	07	
	f -X-	31 84	Generate the sequence category constants a, b, c (integers), and store in R <sub>A</sub> , R <sub>B</sub> , R <sub>C</sub> .  0 ≤ a ≤ 13 0 ≤ b ≤ 16 0 ≤ c ≤ 19  0-19 correspond to R <sub>0</sub> -R <sub>19</sub> via indirect addressing.		RCL C	34 13	if a %a b cor- respond to a sequence component which is a constant or bi- phasic constant = 0, then adjust a %a b to alter the sequence category.  a' = a + 3 b' = b + 3
	h RTN	35 22			g X74	32 81	
	f LBL E	31 25 15			GTO 3	22 03	
	DSP 0	23 00			h S	35 52	
030	RCL B	34 08			RCL B	34 12	
	h RTN	35 22	Generate the sequence category constants a, b, c (integers), and store in R <sub>A</sub> , R <sub>B</sub> , R <sub>C</sub> .  0 ≤ a ≤ 13 0 ≤ b ≤ 16 0 ≤ c ≤ 19  0-19 correspond to R <sub>0</sub> -R <sub>19</sub> via indirect addressing.		g X74	32 81	if a %a b cor- respond to a sequence component which is a constant or bi- phasic constant = 0, then adjust a %a b to alter the sequence category.  a' = a + 3 b' = b + 3
	f LBL B	31 25 12			GTO 5	22 03	
	RCL D	34 14			h SF 2	35 51 02	
	STO E	33 15		090	h S	35 52	
	f LBL A	31 25 11			RCL A	34 11	
	RCL E	34 15	Generate the sequence category constants a, b, c (integers), and store in R <sub>A</sub> , R <sub>B</sub> , R <sub>C</sub> .  0 ≤ a ≤ 13 0 ≤ b ≤ 16 0 ≤ c ≤ 19  0-19 correspond to R <sub>0</sub> -R <sub>19</sub> via indirect addressing.		g X74	32 81	if a %a b cor- respond to a sequence component which is a constant or bi- phasic constant = 0, then adjust a %a b to alter the sequence category.  a' = a + 3 b' = b + 3
	STO D	33 14			GTO 3	22 03	
	f GSB 0	31 22 00			h SF 1	35 51 01	
	1	01			RCL C	34 13	
040	4	04			1	01	
	X	71	Generate the sequence category constants a, b, c (integers), and store in R <sub>A</sub> , R <sub>B</sub> , R <sub>C</sub> .  0 ≤ a ≤ 13 0 ≤ b ≤ 16 0 ≤ c ≤ 19  0-19 correspond to R <sub>0</sub> -R <sub>19</sub> via indirect addressing.		2	02	if a %a b cor- respond to a sequence component which is a constant or bi- phasic constant = 0, then adjust a %a b to alter the sequence category.  a' = a + 3 b' = b + 3
	f INT	31 83			+	61	
	STO A	33 11			STO C	33 13	
	f GSB 0	31 22 00		100	f LBL 3	31 25 03	
	1	01			RCL A	34 11	
	7	07	Generate the sequence category constants a, b, c (integers), and store in R <sub>A</sub> , R <sub>B</sub> , R <sub>C</sub> .  0 ≤ a ≤ 13 0 ≤ b ≤ 16 0 ≤ c ≤ 19  0-19 correspond to R <sub>0</sub> -R <sub>19</sub> via indirect addressing.		h STI	35 33	if a %a b cor- respond to a sequence component which is a constant or bi- phasic constant = 0, then adjust a %a b to alter the sequence category.  a' = a + 3 b' = b + 3
	X	71			RCL (A)	34 24	
	f INT	31 83			f X=0	31 51	
	STO B	33 12			f GSB 5	31 22 05	
050	f GSB 0	31 22 00			h CF 1	35 61 01	
	2	02	Generate the sequence category constants a, b, c (integers), and store in R <sub>A</sub> , R <sub>B</sub> , R <sub>C</sub> .  0 ≤ a ≤ 13 0 ≤ b ≤ 16 0 ≤ c ≤ 19  0-19 correspond to R <sub>0</sub> -R <sub>19</sub> via indirect addressing.		RCL B	34 12	if a %a b cor- respond to a sequence component which is a constant or bi- phasic constant = 0, then adjust a %a b to alter the sequence category.  a' = a + 3 b' = b + 3
	0	00			h STI	35 33	
	X	71			RCL (A)	34 24	
	f INT	31 83		110	f X=0	31 51	
	STO C	33 13			f GSB 6	31 22 06	
	0	00			h CF 2	35 61 02	

## REGISTERS

0	1	1	-1	2	(-1) <sup>n</sup>	3	k <sub>1</sub>	4	k <sub>2</sub>	5	-k <sub>3</sub>	6	(-1) <sup>n</sup> k <sub>2</sub>	7	(-1) <sup>n</sup> k <sub>1</sub>	8	n	9	(-1) <sup>n</sup> n
S <sub>0</sub>	k <sub>3</sub> - n	S <sub>1</sub>	n - k <sub>3</sub>	S <sub>2</sub>	a <sub>n-1</sub>	S <sub>3</sub>	a <sub>n-2</sub>	S <sub>4</sub>	-a <sub>n-1</sub>	S <sub>5</sub>	-a <sub>n-2</sub>	S <sub>6</sub>	k <sub>3</sub> n	S <sub>7</sub>	a <sub>n-1</sub> + a <sub>n-2</sub>	S <sub>8</sub>	a <sub>n-1</sub> - a <sub>n-2</sub>	S <sub>9</sub>	n <sup>2</sup>
A	a on a'		B	b on b'		C	c on c'		D	RAND hold		E	Working RAND		I	Indirect			

STEP	KEY ENTRY	KEY CODE	COMMENTS	STEP	KEY ENTRY	KEY CODE	COMMENTS
	RCL D	34 14	After initialization is complete, display for print the generating seed for later use.		STO 7	23 07	... $(-1)^n \rightarrow R_2$ $(-1)^n k_2 \rightarrow R_6$ $(-1)^n k_1 \rightarrow R_7$ $n \rightarrow R_8$ $(-1)^n n \rightarrow R_9$
	DSP 9	23 09		170	RCL 4	24 04	
	h SPACE	35 84			RCL 2	34 02	
	f -x-	31 84			X	71	
	h SPACE	25 84			STO 6	33 06	
	h RTN	35 22			RCL 2	34 02	
120	f LBL 2	21 25 02	Generate the placeholders for $A_{n-1}$ and $A_{n-2}$ : $-2, -1, -1, 1, 1, 1, 2, 3$ with equal probab. l.		RCL 8	34 08	
	f GSB 1	21 22 01			X	71	
	h ABS	35 64			STO 9	33 09	
	2	02			RCL 5	34 05	
	-	51			CHS	42	
	f x=0	31 51		180	RCL 8	34 08	Update additional components $k_2 - n \rightarrow R_{50}$ $n - k_3 \rightarrow R_{51}$ $k_3 n \rightarrow R_{56}$ $n^2 \rightarrow R_{59}$
	1	01			X	71	
	h RTN	35 22			f P $\rightarrow$ S	31 42	
	f LBL 1	31 25 01	Generate a random integer $-3 \leq n \leq 5$ for $k_1, k_2, k_3$ (+ modified for $A_{n-1}, A_{n-2}$ placeholders.)		STO 6	33 06	
	f GSB 0	31 22 00			h LST x	35 82	
130	9	09			9 x <sup>2</sup>	22 54	
	X	71			STO 9	33 09	
	f INT	31 83			1	01	
	3	03			STO + 1	33 61 01	
	-	51			STO - 0	33 51 00	
	h RTN	35 22		190	RCL 2	24 02	Complete up- dating of sequence components $-A_{n-1} \rightarrow R_{54}$ $-A_{n-2} \rightarrow R_{55}$ $A_{n-1} + A_{n-2} \rightarrow R_{57}$ $A_{n-1} \cdot A_{n-2} \rightarrow R_{58}$
	f LBL 0	31 25 00	Generate random decimal d $0 \leq d \leq 1$ $RND_n = \text{fract part}$ of $(RND_{n-1} + \pi)^2$		STO 7	33 07	
	RCL E	34 15			STO 8	33 08	
	h $\pi$	35 73			CHS	42	
	+	61			STO 4	33 04	
140	ENTER $\uparrow$	41			RCL 3	34 03	
	X	71			STO + 7	33 61 07	
	9 fract	32 83			STO - 8	33 51 08	
	STO E	33 15			CHS	42	
	h RTN	35 22		200	STO 5	33 05	Generate $A_n$ , the next term of the sequence: $A_n = (a) \cdot (b) + (c)$ or $(a') \cdot (b') + (c')$ where $(a')$ refers to sequence compo- nent in $R_{54}$ etc.
	f LBL 5	31 25 05	For $\begin{cases} a \leq 7 \\ R_a = 0 \end{cases}$ Let $a' = a + 3$ replace a in $R_A$ .		f P $\rightarrow$ S	31 42	
	0	00			RCL A	34 11	
	h F? 1	35 71 01			h STI	35 33	
	3	03			RCL (a)	34 24	
	RCL A	34 11			RCL B	34 12	
	+	61			h STI	35 33	
150	STO A	33 11			h $\pi$	35 52	
	h RTN	35 22			RCL (a)	34 24	
	f LBL 6	31 25 06	For $\begin{cases} b \leq 7 \\ R_b = 0 \end{cases}$ Let $b' = b + 3$ replace b in $R_B$ .		X	71	Update sequence terms $A_{n-1} + A_{n-2}$ for later use.
	0	00			RCL C	34 13	
	h F? 2	35 71 02		210	h STI	25 33	
	3	03			h $\pi$	35 52	
	RCL B	34 12			RCL (b)	34 24	
	+	61			+	61	
	STO B	33 12			f P $\rightarrow$ S	31 42	
	h RTN	35 22			RCL 2	34 02	
160	f LBL D	21 25 14	Generate next sequence element.  Start by up- dating components ...		STO 3	33 03	
	1	01			h $\pi$	35 52	
	STO + 8	33 61 08			STO 2	33 02	
	CHS	42			f P $\rightarrow$ S	31 42	
	RCL 8	34 08		220	DSP 0	23 00	Display/print $A_n$ and halt - unless auto mode is set.
	h y <sup>x</sup>	35 63			f -x-	31 84	
	STO 2	33 02			h F? 0	35 71 00	
	RCL 3	34 03			STO 0	22 14	
	X	71			h RTN	35 22	

## LABELS

## FLAGS

## SET STATUS

A NEW S	B LAST S	C	D GO	E SEE N	0 AUTO ?	FLAGS	TRIG	DISP
a AND $\Phi$	b	c	d AUTO ?	e SEE a bc	1 a' for a	ON OFF	DEG <input checked="" type="checkbox"/>	FIX <input checked="" type="checkbox"/>
0 CONTIN. RAND   0	1 INTEGER RAND   5	2 placeholders	3 after c' for c	4 AUTO	2 b' for b	0 <input type="checkbox"/> <input checked="" type="checkbox"/>	GRAD <input type="checkbox"/>	SCI <input type="checkbox"/>
5 a' adjust	6 b' adjust	7	8	9	3 new AND ?	1 <input type="checkbox"/> <input checked="" type="checkbox"/>	RAD <input type="checkbox"/>	ENG <input type="checkbox"/>
						2 <input type="checkbox"/> <input checked="" type="checkbox"/>		n 2
						3 <input type="checkbox"/> <input checked="" type="checkbox"/>		